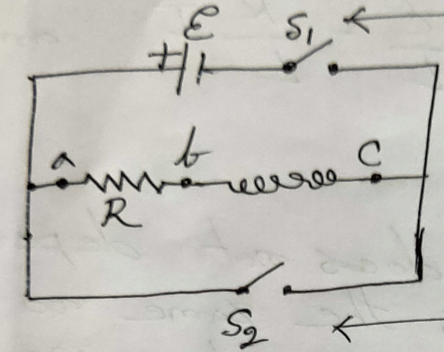
L-R Circuit and Current Growth in in an L-R Circuit

A Circuit that includes both a resistor and an inductor, and possibly a Source of emf, is called an R-L Circuit.



closing S_1 connects the R-L Combination in series with a source of emf $\mathcal{E}$.

closing Swith S_2 While opening Swith S_1 disconnects the combination from the Source.

fig 1:- An R-L Circuit

Let i be the current at some time t after switch S_1 is closed and let di/dt be its rate of change at that time. The potential differences V_{ab} across the resistor and V_{bc} across the inductor are

$$V_{ab} = iR \quad \text{and} \quad V_{bc} = L\frac{di}{dt}$$

then according to kirchhoff's loop rule.

$$\mathcal{E} - iR - L\frac{di}{dt} = 0 \quad \text{———} \quad (1)$$

Solving this for $\frac{di}{dt}$, we find that the rate of increase of current is

$$\frac{di}{dt} = \frac{\mathcal{E} - iR}{L} = \frac{\mathcal{E}}{L} - \frac{R}{L}i \quad \text{———} \quad (2)$$

If Switch S_1 is 1st closed, $i=0$ & potential drop across R is zero.
The initial rate of change of current is

$$\left(\frac{di}{dt}\right)_{initial} = \frac{\mathcal{E}}{L}$$

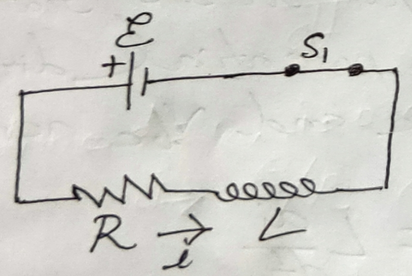
The greater the inductance L, the more slowly the current increases.

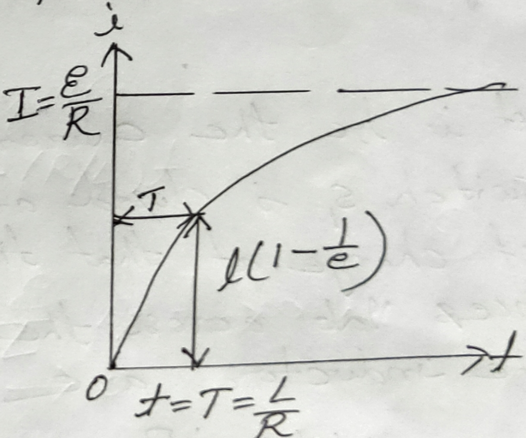
$$\left(\frac{di}{dt}\right)_{final} = 0 = \frac{\mathcal{E}}{L} - \frac{R}{L}I \quad \text{and}$$

$$I = \frac{\mathcal{E}}{R}.$$

The final current I does not depend on the inductance L. It is the same as it would be if the resistance R alone were connected to the source with emf $\mathcal{E}$.

Switch S_1 is closed at $t=0$





Graph of i versus t for growth of current in an R-L circuit with an emf in series. The final current $I = \mathcal{E}/R$; after one time constant T, the current is $1-\frac{1}{e}$ of this value.

Fig2 :- Behavior of the current as a function of time

After Rearranging eqn ② we have

$$\frac{di}{i-(\mathcal{E}/R)} = -\frac{R}{L}dt$$

$$\int_0^i \frac{di'}{i'-(\mathcal{E}/R)} = -\int_0^t \frac{R}{L}\,dt'$$

$$\ln\left(\frac{i-(\mathcal{E}/R)}{-\mathcal{E}/R}\right) = -\frac{R}{L}t$$

Now we take exponentials of both sides and solve for i.

$$i = \frac{\mathcal{E}}{R}\left(1-e^{-(R/L)t}\right) \qquad \text{③}$$

where i = current in an R-L circuit with emf.

Taking the derivative of eqn ③ we have

$$\frac{di}{dt} = \frac{\mathcal{E}}{L}e^{-(R/L)t} \qquad \text{④}$$

At time $t=0$, $i=0$ and $\frac{di}{dt} = \frac{\mathcal{E}}{L}$. As $t \to \infty$,
$i \to \mathcal{E}/R$ and $\frac{di}{dt} \to 0$

Time constant $\tau = \frac{L}{R}$ ⑤ [where L = Inductance, R = Resistance]

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